ASEN 5327 Computational Fluid Dynamics
Spring 2009

Homework 4, due Tuesday, April 7

1. Derive the flux Jacobian matrix $\partial E/\partial U$ for the two-dimensional Euler equations. The vectors E and U are defined as

$$U = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ E_t \end{bmatrix} \qquad E = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ (E_t + p)u \end{bmatrix}$$

where the total internal energy is

$$E_t = \rho(e + 1/2(u^2 + v^2)) = \rho(c_v T + 1/2(u^2 + v^2)) = \frac{p}{\gamma - 1} + 1/2\rho(u^2 + v^2)$$

and where $\gamma \equiv c_p/c_v$ is the ratio of specific heats. The ideal gas law, expression for the speed of sound, and connection between R c_p and c_v are

$$p = \rho RT, \qquad c^2 = \frac{\gamma p}{\rho}, \qquad R = c_p - c_v.$$

Extra credit - Derive (or at least show convincingly) that the eigenvalues of $\partial E/\partial U$ are

$$u, \quad u, \quad u + c, \quad u - c.$$

2. Write a computer program to solve the diffusion equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$$

on a square domain $0 \le x \le 1, \quad 0 \le y \le 1$, subject to the following Dirchlet boundary conditions

$$u(0, y, t) = 0.0, \qquad u(1, y, t) = 1.0, \qquad u(x, 0, t) = x, \qquad u(x, 1, t) = \sin(\pi(0.5+2n)x)$$

where n is a parameter to be specified below. The initial condition is not terribly important as we are seeking the steady-state solution. It is convenient to simply set $u = 0$ initially on the interior. Use central differences in space and the time advancement scheme of your choice. Compute in double precision, and iterate until the rms difference between the solution at two successive time levels is below 1.0×10^{-7}. This may take several hundred thousand iterations on a large mesh. For each case below, plot the final solution along the line $y = 0.96$.

(a) Use a 50x50 mesh with n=4.

(b) Use a 100x100 mesh with n=4.

(c) Use a 200x200 mesh with n=8.

(d) Use a 500x500 mesh with n=8.

Finally, produce a table where the number of iterations and computer time required for each case is displayed.