# ASEN 5327 Computational Fluid Dynamics
# Spring 2009

## Homework 5, due Thursday, April 16

The quasi-one-dimensional Euler equations can be written as

$$\frac{\partial}{\partial t}(\vec{U}A) + \frac{\partial}{\partial x}(\vec{F}A) = \vec{Q} \qquad (1)$$

where

$$\vec{U} = \left\{ \begin{array}{c} \rho \\ \rho u \\ E_t \end{array} \right\} \qquad \vec{F} = \left\{ \begin{array}{c} \rho u \\ \rho u^2 + p \\ (E_t + p)u \end{array} \right\} \qquad \vec{Q} = \left\{ \begin{array}{c} 0 \\ p\frac{dA}{dx} \\ 0 \end{array} \right\} \qquad (2)$$

and where $A(x)$ is the cross-sectional area.

Write a computer program to solve the quasi-one-dimensional Euler equations for the flow through a supersonic inlet, whose cross-sectional area distribution is given as

$$A(x) = A_t\{1 + 4[(x/L) - 1/2]^2\}; \qquad \text{for } 0 \le x/L \le 1 \qquad (3)$$

where $A_t = \pi 0.05^2\ m^2$ and $L = 0.6\ m$. As inlet conditions, use $p = 1.0\ atm$, $T = 288\ K$, $M = 2.3$. For downstream condition, use $p = 9.0\ atm$. Start with a mesh containing 50 intervals and initialize the solution with the isentropic steady quasi-one-dimensional result. This initial condition will not satisfy the given downstream boundary condition, and this mismatch should propagate upstream as a shock wave, which will become stationary when a steady-state is achieved.

Use the explicit MacCormack method as discussed in Section 4.4.3. Compare your computed solution with the exact quasi-one-dimensional solution that includes a shock in order match the downstream condition. Compute the rms difference between your computed pressure and the exact pressure and produce a convergence plot where the error is plotted as a function of the number of mesh points for meshes containing 50, 100, 200, 400, and 800 points. Also plot the computed pressure against the exact pressure for each case. You may also want to turn in any derivations that you went through in order to write the code.