

**Practice Final Exam**

The final exam will cover material from the entire course. The practice problems below are for the second half of the semester only. You can use the practice midterm as well as the midterm exam to review the first half of the course. The format of the final exam will be identical to the midterm, i.e. multiple choice. One change is that you may turn in the work you did in order to arrive at your answers. I will look at this if you answer a problem incorrectly to see if any partial credit is warranted. I can only give partial credit if your solution is clearly worked out and I can see exactly where you made a mistake. Do not expect to receive partial credit for jumbled, incoherent derivations. The exam is open to books and notes, but closed to any sort of electronic devices.

1.

- (a) Write down the integral form of the energy conservation law.
- (b) Is this law in conservative form? If not, indicate what modification must be made in order to put it in conservative form.
- (c) Write down a discrete form of the energy conservation law in two dimensions, where the midpoint rule is used to evaluate the integrals. Use a notation like  $u_{i,j}^e$  to denote a quantity interpolated to the east face of the cell  $i, j$ .
- (d) Explain why or why not it is possible to time advance this discrete equation with a backward bias on the flux interpolation for a case where the velocity is positive.

2.

- (a) Write down a physically-consistent, first order finite volume approximation to Burgers equation. Leave the time derivative term in continuous form and indicate clearly the form of any interpolation operators used. Use a notation like  $u_i$  to indicate solution values at cell centers. Do not assume a uniform mesh.
- (b) Explain what must be done to this scheme in order to increase the order of accuracy (but do not actually derive such a scheme).
- (c) Explain how the interpolations must be done if MacCormack's scheme is to be used.

3.

- (a) What are the eigenvalues of the  $x$ -derivative terms for the 2D Euler equations?
- (b) Under what conditions (if any) are all of the eigenvalues positive?
- (c) Under what conditions (if any) is one eigenvalues negative and the rest positive?
- (d) Under what conditions (if any) are two eigenvalues negative and the rest positive?
- (e) Under what conditions (if any) are three eigenvalues negative and the rest positive?

4. Answer the following for the 2D Euler equations.

- (a) What characteristic relations (if any) must be solved at a supersonic inlet boundary?
- (b) What characteristic relations (if any) must be solved at a subsonic exit boundary?
- (c) What characteristic relations (if any) must be solved at a solid wall boundary?

5. Consider an "East" cell face in two dimensions with endpoints  $(x_{i+1,j}, y_{i+1,j})$  and  $(x_{i+1,j+1}, y_{i+1,j+1})$

- (a) Write down the expressions for the components of the "area weighted normal vector,"  $\vec{n}\Delta s$ . Use a sign convention where the normal points in the direction of increasing  $i$  index.
- (b) How must these normal components be modified if they are to be used for the West face for the cell  $i + 1, j$ ?
- (c) Explain why it is convenient to work with the "area weighted normal vector."