ASEN 5327 Computational Fluid Dynamics

Answers to Practice Final Exam

1.

(a)

$$\frac{d}{dt}\int_V E_t \, dV + \int_S (E_t + p)(\vec{u}\cdot\vec{n}) \, dS = 0$$

(b) The above relation is in conservation law form. If the volume is taken as the whole domain, then the second integral represents the fluxes on the boundaries of the domain, which means there is no opportunity for ficticious source terms on the interior.

(c)

$$\frac{dE_{ti,j}}{dt} = -\frac{1}{V_{i,j}} \sum_{l=n,e,s,w} (E_t + p)^l_{i,j} (\vec{u}\cdot\vec{n}\Delta S)^l_{i,j}$$

(d) In general, it is not possible to advance the energy equation with a consistent backward bias on the flux interpolations. This is because the energy equation is coupled to the other equations, whoes overall eigenvalue system usually has eigenvalues of both signs. We must respect the domains of dependence implied by the signs of the eigenvalues, which means some sort of split (or alternating) bias on the flux interpolations.

2.

(a)

$$\frac{du_i}{dt} = -\frac{1}{2\Delta x_i}[(u_i^e)^2 - (u_i^w)^2]$$

where

$$u_i^e = \begin{cases} u_i & \text{for } u_i > 0 \\ u_{i+1} & \text{for } u_i < 0 \end{cases} \qquad u_i^w = \begin{cases} u_{i-1} & \text{for } u_i > 0 \\ u_i & \text{for } u_i < 0 \end{cases}$$

(b) Higher order interpolations can be used to increase the accuracy of the scheme. For example, the solution values u_{i+2}, u_{i+1}, and u_i can be used to determine u_i^e when $u_i < 0$. Polynomial fitting can be used to determine the corresponding interpolation weights.

(c) If MacCormack's scheme is to be used, then first order interpolations are used, but with an alternating bias between the predictor and corrector steps. For example, $u_i^e = u_{i+1}$ on the predictor step and $u_i^e = u_i$ on the corrector step.

3.

(a) The eigenvalues for the x-derivative terms in the 2D Euler equations are u, u, $u+c$, and $u-c$, where c is the speed of sound.

(b) All of the eigenvalues will be positive if $|u| > c$ and $u > 0$.

(c) One eigenvalue will be negative and the rest positive if $|u| < c$ and $u > 0$.

(d) It is not possible to have two positive and two negative eigenvalues.

(e) One eigenvalue will be positive and the rest negative if $|u| < c$ and $u < 0$.

4.

(a) All four characteristic directions are positive for a supersonic inlet boundary and thus we can specify the entire solution at this boundary without solving any characteristic relations.

(b) At a subsonic exit boundary, three characteristic directions are positive and one negative. Thus we must solve the following three characteristic equations, which correspond to the positive eigenvalues.

$$\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x}\right) - \frac{1}{c^2}\left(\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x}\right) = 0 \tag{1}$$

$$\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x}\right) = 0 \tag{2}$$

$$\left(\frac{\partial u}{\partial t} + (u+c)\frac{\partial u}{\partial x}\right) + \frac{1}{\rho c}\left(\frac{\partial p}{\partial t} + (u+c)\frac{\partial p}{\partial x}\right) = 0 \tag{3}$$

$$\tag{4}$$

(c) There are no fluxes across a solid boundary. Thus we do not solve any characteristic relations in this case.

5.

(a)

$$n_x \Delta S = (y_{i+1,j+1} - y_{i+1,j})\Delta z \qquad n_y \Delta S = -(x_{i+1,j+1} - x_{i+1,j})\Delta z$$

(b) The signs of both $n_x\Delta S$ and $n_y \Delta S$ must be changed, i.e.

$$n_x \Delta S = -(y_{i+1,j+1} - y_{i+1,j})\Delta z \qquad n_y \Delta S = (x_{i+1,j+1} - x_{i+1,j})\Delta z$$

(c) It is convenient to work with the "area weighted normal vector" since it is this quantity that enters the flux balance, i.e. $F n_x \Delta S + G n_y \Delta S$